# TEMPERATURE OF THE SURFACE OF A BODY UNDERGOING DESTRUCTION BY A CONSTANT THERMAL LOAD

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A relation is proposed for calculating the temperature of a disintegrating surface in a transient period. A connection is established between the constant $K_{\tau_d}$ and the law of change of the surface temperature. Equations are obtained to determine constants of the process of heating and destruction of the material.

When the surface of a material is heated by a constant heat flow, two laws of change in the temperature of the surface are possible. If the heat flux is inadequate to attain the disintegration temperature $T_d$, then the surface temperature increases in accordance with the law: $T_w = d\tau^{1/2}$ [1]. At higher rates of heating, after a certain time $\tau_{T_d}$ the temperature may reach $T_d$ and, with further heating, a quasisteady temperature $\bar{T}_w$ may be attained on the surface at the moment of time $\tau_T$, i.e., in a first approximation it can be assumed that $T_w$ = const [2].

It was shown in [3] that if the temperature of the surface of a body heated without undergoing disintegration changes by the law

$$T_w(\tau) = \sum_{n=0}^{N} d_n \tau^{n/2},$$

then the temperature field inside the body, with constant thermophysical properties, will have the form

$$T(y, \tau) - T_0 = \sum_{n=0}^{N} d_n \tau^{n/2} 2^{n/2} \Gamma(n/2) I^n \operatorname{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right). \tag{1}$$

Having designated $y/\sqrt{a\tau}$ through K, for the laws $T_w$ = const and $T_w = d\tau^{1/2}$, we respectively obtain:

$$\Theta^* = \operatorname{erfc}\frac{K}{2}, \tag{2}$$

$$\Theta^* = \exp\left[-\left(\frac{K}{2}\right)^2\right] - \frac{K}{2}\sqrt{\pi}\operatorname{erfc}\frac{K}{2}, \tag{3}$$

where $\Theta^* = (T^* - T_0)/(T_w - T_0)$.

It was noted in [4, 5] that despite the fact that the coefficient K includes an isotherm coordinate, diffusivity, and heating time, it is independent of them and is determined only by the law of change in $T_w$ and $\Theta^*$. It was established in [4] that with the entrainment of material from the surface, Eq. (2) can be applied at $\Theta^* < 0.2$. At $\Theta^* > 0.2$, the temperature coefficient K is found from the formula

$$K = -\frac{1}{K_{\tau_d}}\Theta^* + \frac{K_{\tau_d}^2}{1 - K_{\tau_d}}. \tag{4}$$

Here, in contrast to (2) and (3) — where $\Theta^* = 1$ and $K = 0$ — in the present case $K = K_{\tau_d} \approx 0.74$. It is evident from Eqs. (2) and (3) that with a prescribed values of $\Theta^*$, the coefficient is greater, the slower the change in $T_w$. It is not hard to show that a reduction in the exponent in the law $T_w = d\tau^{n/2}$ also leads to an increase in the amount of heat absorbed by the body. During the period of time from $\tau_{T_d}$ to $\tau_T$, surface temperature increases from $T_d$ to $\bar{T}_w$ due to the accumulation of heat in the layer of material undergoing disintegration (Fig. 1). Since Eq. (4) is valid only for disintegration of the surface at $T_w$ = const, it can be

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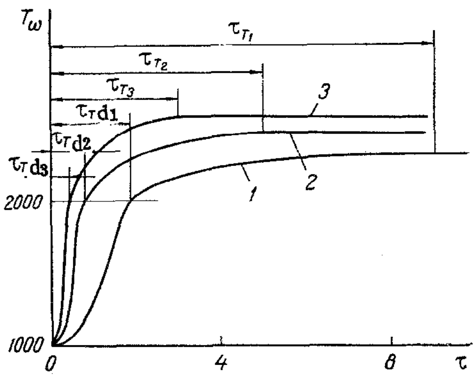

Fig. 1. Dependence of the surface temperature of materials based on quartz glass on the heating time in a gas-generator jet at $q_x$ = 14,700 kW/m²: 1) quartz glass; 2) pure glass-ceramic; 3) alloyed glass-ceramic.

suggested that the temperature coefficient for $\Theta^* = 1$ is equal to $K_{T_d}$ from the moment of time $\tau_T$. It is interesting to establish the law of change in $T_w$ during the period from $\tau_{T_d}$ to $\tau_T$ and its relationship with the constant $K_{T_d}$.

Allowing for the above, we used the following ratio to analyze oscillograms of surface temperature in more than 200 experiments involving the heating of specimens of quartz glass and pure and alloyed glass-ceramics

$$\frac{T_w - T_d}{\bar{T}_w - T_d} = \Theta_{T_w} = f(t). \tag{5}$$

Here, $t = (\sqrt{\tau} - \sqrt{\tau_{T_d}})/(\sqrt{\tau_T} - \sqrt{\tau_{T_d}})$, where $\tau_{T_d}$ is the time required to reach the temperature at which a melt of the quartz glass-ceramic forms, ~2000°K [6]. We used values of emissivity from [6] when we analyzed the measurements. The method and accuracy of the determination of surface temperature were described in the same study. The times $\tau_{T_d}$ and $\tau_T$ were found from the oscillograms (Fig. 1). Here, the time $\tau_T$ corresponded to the establishment of the mean temperature of the surface:

$$\bar{T}_w = \int\limits_{\tau_T}^{\tau} T_w d\tau/(\tau - \tau_T).$$

Table 1 shows values of $\tau_{T_d}$ and $\tau_T$ determined for different heating regimes and obtained from 15-20 experiments.

Figure 2 shows certain temperature relations analyzed in the form of Eq. (5). These results are described by a fifth-degree polynomial with a standard deviation of 0.06 for the test data:

$$\Theta_{T_w} = 0{,}233t^5 + 0{,}754t^4 - 0{,}838t^3 + 0{,}919t^2 - 0{,}104t + 0{,}019. \tag{6}$$

However, in our case it is desirable to find the function (5), which includes $K_{T_d}$. It is evident from Fig. 2 that nearly all of the experimental results lie below the curve corresponding to the equation for a half-circle of unit radius

$$\Theta_{T_w} = \sqrt{t(2-t)}. \tag{7}$$

At the same time, the experimental data is bounded from below by a curve which satisfies the expression

$$\Theta_{T_w} = (1 + K_{T_d}^2) - \frac{K_{T_d}^3 \sqrt{10^7}}{2\sqrt{\tau}(\bar{T}_w - T_d)}. \tag{8}$$

Equation (8) is analogous to the rate equation obtained in [7]. Here, the constant $10^7$ has the dimension K²·sec.

We obtain the following from (8) for the time $\tau_{T_d}$ at $\Theta_{T_w} = 0$:

$$\tau_{T_d} = \frac{K_{T_d}^6 \cdot 10^7}{4(1 + K_{T_d}^2)^2(\bar{T}_w - T_d)^2}, \tag{9}$$

TABLE 1. Experimental Values of the Time Required to Reach the Surface Temperature Corresponding to the Beginning of Disintegration and the Quasisteady Value

| Material | $q_x$, kW/m² | $I_e$, kJ/kg | $T_w$, K | $\tau_{T_d}$, sec | $\tau_T$, sec |
|---|---|---|---|---|---|
| Alloyed quartz glass-ceramic (0,5% $Cr_2O_3$) | 5850 | 7200 | 2490 | 0,87 | 5,7 |
| | 7650 | 8600 | 2620 | 0,50 | 3,9 |
| | 11500 | 12300 | 2800 | 0,25 | 2,5 |
| | 14700 | 4700 | 2600 | 0,42 | 2,5 |
| Pure glass-ceramic (100% $SiO_2$) | 5850 | 7200 | 2480 | 1,21 | 5,4 |
| | 7650 | 8600 | 2610 | 0,62 | 4,0 |
| | 11500 | 12300 | 2710 | 0,3 | 2,6 |
| | 14700 | 4700 | 2490 | 0,78 | 5,0 |
| Quartz glass | 5850 | 7200 | 2370 | 2,25 | 8,4 |
| | 7650 | 8600 | 2590 | 1,13 | 5,4 |
| | 11500 | 12300 | 2660 | 0,53 | 3,5 |
| | 14700 | 4700 | 2360 | 1,76 | 9,0 |

while at $\Theta_{T_w} = 1$ we have

$$\tau_T = \frac{K_{T_d}^2 \cdot 10^7}{4(\bar{T}_w - T_d)^2}. \tag{10}$$

Experimental values of $\tau_T$ shown in Table 1 agree satisfactorily with the results calculated from (10) (Fig. 3).

We can use Eqs. (7) and (8) to find an expression for the surface temperature of a body undergoing disintegration under the influence of a constant thermal load:

$$\Theta_{T_w} = \frac{1}{2}\left\{t(2-t) + 2\left[(1+K_{T_d}^2) - \frac{K_{T_d}^3 \sqrt{10^7}}{2\sqrt{\tau}(\bar{T}_w - T_d)}\right] - \left[(1+K_{T_d}^2) - \frac{K_{T_d}^3 \sqrt{10^7}}{2\sqrt{\tau}(\bar{T}_w - T_d)}\right]^2\right\}, \tag{11}$$

where $\tau$ changes from $\tau_{T_d}$ to $\tau_T$, determined by (9) and (10). The standard deviation of the experimental results from Eq. (11), as in the case of calculations with Eq. (6), is 0,06.

Integrating (11), we find that

$$\int_0^1 \Theta_{T_w} dt = 0{,}742 \approx K_{T_d}. \tag{12}$$

It is evident from this that the law of change in surface temperature during the period of time from $\tau_{T_d}$ to $\tau_T$ determines the constant $K_{T_d}$.

We can use the equation proposed in [4, 7] for determining $K_{T_d}$

$$2K_{T_d}^3 - K_{T_d}^2 + K_{T_d} - 1 = 0 \tag{13}$$

to obtain several identities. One of these identities

$$\frac{K_{T_d}^2 + 1}{K_{T_d}} = \frac{K_{T_d}^2}{1 - K_{T_d}} \tag{14}$$

makes it possible to simplify Eq. (4) and to write

$$K = \frac{1}{K_{T_d}}(1 - \Theta^*) + K_{T_d}. \tag{15}$$

The identity

$$\frac{K_{T_d}}{(K_{T_d}^2 + 1)(1 - K_{T_d})} = \frac{1}{K_{T_d}^2} = 1{,}831 \tag{16}$$

establishes the relationship between the rate on the isotherm $\Theta^* = 1$ and the rate of disintegration of the surface [7]. Of equal interest is the relationship

$$\frac{K_{T_d}^2 + 1}{K_{T_d}^2} = \frac{K_{T_d}}{1 - K_{T_d}} = 2{,}831, \tag{17}$$

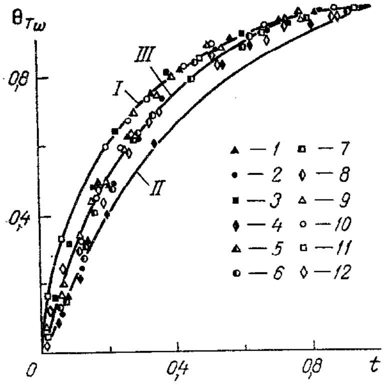

Fig. 2

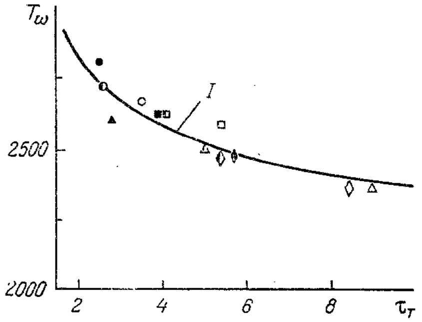

Fig. 3

Fig. 2. Change in the relative temperature gradient in a disintegrating layer of material $\Theta_{T_w}$ vs the dimensionless heating time t: 1-4) alloyed quartz glass-ceramic; 5-8) pure glass-ceramic; 9-12) quartz glass; 1, 5, 9) $q_x$ = 14,700 kW/m², $P_e$ = $3.5 \cdot 10^5$ Pa; 2, 6, 10) $q_x$ = 11,500 kW/m²; 3, 7, 11) $q_x$ = 7650 kW/m²; 4, 8, 12) $q_x$ = 5850 kW/m² (all $P_e \approx 10^5$ Pa); I-III) calculation from Eqs. (7), (8), and (11).

Fig. 3. Dependence of the quasisteady value of surface temperature on the time required to establish it: I) calculation from (10); points denote experimental results (the notation is the same as in Fig. 2).

since it links the time ratios

$$\frac{\sqrt{\tau_v}}{\sqrt{\tau_a}} = \frac{\sqrt{\tau_T}}{\sqrt{\tau_{T_d}}} = \frac{K^2_{T_d}+1}{K^2_{T_d}}.$$

Finally, the fourth identity

$$\frac{1+K^2_{T_d}}{K^3_{T_d}} = \frac{1}{1-K_{T_d}} = 3{,}831 \tag{18}$$

determines the constant 6.79:

$$6{,}79 = \frac{(1+K^2_{T_d})\sqrt{\pi}}{K^3_{T_d}} = \frac{\sqrt{\pi}}{1-K_{T_d}}. \tag{19}$$

The constant entering into the formula for calculation of the heat of vaporization, found in [8] and equal to half the value of the constant determined by Eq. (19), can also be represented in the form

$$3{,}4 \approx \sqrt{2}\frac{\sqrt{\pi}}{K_{T_d}}. \tag{20}$$

The constants (19) and (20) establish the relationship between the heat content of the material on the one hand and its total heat of destruction H and heat of vaporization $\Delta Q_v$ [9] on the other hand.

It was shown above that the change in the temperature of the surface of a material subjected directly to an external thermal load can be described in a first approximation by the equation for a half-circle of unit radius (7) (see Fig. 2). It is interesting that the ratio of the ordinate of the point on such a half-circle at t = $K_{T_d}$ to the length of its arc up to this point is equal to the disintegration constant (Fig. 4), i.e.,

$$y_x/l_x = K_{T_d}. \tag{21}$$

Here

$$y_x = \sqrt{K_{T_d}(2-K_{T_d})} = 0{,}965, \tag{22}$$

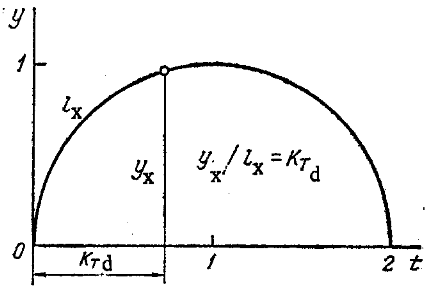

Fig. 4. Relationship between the constant $K_{T_d}$ and elements of a unit circle.

while

$$l_x = \operatorname{arctg} \frac{\sqrt{K_{T_d}(2 - K_{T_d})}}{1 - K_{T_d}} = 1{,}307. \tag{23}$$

It is not hard to use these relations to establish the connection between $K_{T_d}$ and the number $\pi$. We obtain the following from Eqs. (21) and (22)

$$K_{T_d} + y_x l_x = 2. \tag{24}$$

Inserting (21) into (24), we find that

$$l_x = \sqrt{\frac{2 - K_{T_d}}{K_{T_d}}}. \tag{25}$$

It can then be shown that the ratio $l_x/(\pi/2) = 0.832$, while since $1/K_{T_d}^2 = 1.831$, then

$$\frac{l_x}{\pi/2} = \frac{1}{K_{T_d}^2} - 1. \tag{26}$$

Having inserted (25) into (26), we find that the number $\pi$ is connected with $K_{T_d}$ by the relation

$$\pi \approx 2 \sqrt{\frac{2 - K_{T_d}}{K_{T_d}}} \frac{K_{T_d}^2}{1 - K_{T_d}^2}, \tag{27}$$

which is satisfied to within 0.05%.

## NOTATION

$T_w$, $T_0$, temperature of heated surface and unheated material; $\overline{T}_w$, quasisteady (mean) surface temperature; $T_d$, temperature of beginning of disintegration of surface; $T^*$, $y$, temperature and coordinate of the isotherm under consideration; $d_n$, proportionality factor in the n-th law of surface-temperature change; $\Gamma$, gamma function; $\tau$, heating time; $\tau_{T_d}$, $\tau_T$, time required to reach the surface temperature corresponding to the beginning of disintegration and the quasisteady value; $K$, temperature coefficient; $K_{T_d}$, disintegration constant; $\tau_a$, $\tau_v$, times of beginning of ablation and establishment of quasisteady rate of disintegration.

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